EFFLUX TIME FOR A CYLINDRICAL TANK

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Introduction

Consider the simple apparatus depicted in the figure to the right; a cylindrical tank filled with water is to be drained through a brass tube. The cylindrical tank has a diameter of about 15.5 cm and it is equipped with a set of discharge tubes and an orifice. We want to observe the change in depth of water with time by measuring *h(t)*. Let's formulate a mass balance for this process:

-[rate out] = [accumulation], or

$$-\pi R^2 \rho V = \pi R_T{}^2 \rho \frac{dh}{dt}, \qquad (1)$$

where *R* and *V* are the radius of the discharge tube and the average velocity in the tube, respectively. We can rewrite the equation in a more convenient form:

$$\frac{dh}{dt} = -\frac{R^2}{R_T{}^2} V \quad . \qquad (2)$$

Therefore, if we can relate *V* to *h* we should, in principle, be able to predict *h(t)* for any drain tube. Now examine the macroscopic mechanical energy balance on page 207 of *Transport Phenomena*. If we neglect the change in kinetic energy and omit the loss factor for the contraction, we are left with:

$$g(z_2 - z_1) = -\tfrac{1}{2} V^2 \frac{L}{R_h} f \ , \qquad (3)$$

where *L* is the length of the drain tube attached to the bottom of the tank. Clearly, we can rewrite (3) to yield:

$$\frac{g(L+h)R}{L} = V^2 f \tag{4}$$

Next, consider the Moody chart on page 182 of your text. If the Reynolds number of the discharge flow falls between about 3000 and 10^5, and if the brass tube is hydraulically smooth, then

$$f \cong \frac{0.0791}{\mathrm{Re}^{1/4}}. \tag{5}$$

We introduce this approximation into (4) and isolate *V*. You should verify that:

$$V = \left[\frac{g(L+h)R^{5/4}}{0.0791L} \left(\frac{2\rho}{\mu} \right)^{1/4} \right]^{4/7} \tag{6}$$

Now our model is ready:

$$\frac{dh}{dt} = -\frac{R^2}{R_T{}^2} \left[\frac{g(L+h)R^{5/4}}{0.0791L} \left(\frac{2\rho}{\mu} \right)^{1/4} \right]^{4/7} \tag{7}$$

We could solve this elementary differential equation analytically, but that would be a mistake for reasons that will soon be apparent. Suppose we discretize the equation by writing:

$$\frac{h_{t+\Delta t} - h_t}{\Delta t} \cong -\frac{R^2}{R_T{}^2} \left[\frac{g(L+h)R^{5/4}}{0.0791L} \left(\frac{2\rho}{\mu} \right)^{1/4} \right]^{4/7} \tag{8}$$

If we multiply by Δt, and add h_t to both sides, we have the Euler method (for numerical solution of the differential equation). Remember that the Euler method is a straight-line piecewise approximation, so if the solution exhibits a lot of curvature we'll need a very small Δt! We'll come back to our discussion of a solution procedure later. There are two other phenomena requiring our attention; we need to assess whether or not they will adversely affect accord between the experimental data and our model.

Entrance Effect

Now consult page 52 of your text, part (e). Very near the entrance to the drain tube, the velocity profile is nearly flat in the center with a very steep decrease at the wall. In this "entrance" region the frictional resistance per unit length is *much* larger than it is for fully developed flow. Consequently, our estimate of the friction factor, *f*, will not yield very good results—particularly if the drain tube we've selected is short. For laminar flow, for example,

$$L_e \approx 0.035d \cdot \mathrm{Re}. \tag{9}$$

So if Re=2000, we'll need *70 tube diameters* to attain the expected parabolic velocity profile. For turbulent flow,

$$L_e \approx 40d\ ; \tag{10}$$

the entrance length is less dependent upon Reynolds number. Therefore, if we use the one of the 0.475 cm diameter tubes, nearly 20 cm of length will be required for development of the velocity distribution. For a short tube, this will have a profound impact upon the results because τ_0 is much larger in the entrance region than it is for fully developed flow.

Acceleration in Startup

And there is another problem—see Figure 4D.2 on page 150 of your text. When we unplug the drain tube, the water within will accelerate for some period of time. Remember, we're using a pseudo-steady state approach to this problem. How long will it take for the water to attain a velocity of, say, 90% of its ultimate centerline value? For *laminar flows* we can estimate the acceleration period using 4D.2. The figure indicates that the centerline velocity will be 90% of the ultimate (maximum) value if $\nu t/R^2 \approx 0.42$. Convince yourself that this t is not a significant fraction of the efflux time of the tank. Under what circumstances might it be?

Draining the Tank through the Orifice

The preceding model for a drain tube of length, L, is not valid for the orifice. You may recall from physics that the velocity of discharge through a hole in a tank can be described with Torricelli's theorem:

$$V = \sqrt{2gh}\ . \tag{11}$$

However, this is a *frictionless* result and we can expect the velocity obtained with it to be too large. A simple fix can be developed using (7.5-10) from *Transport Phenomena*:

$$\tfrac{1}{2}V^2 - gh = -\tfrac{1}{2}V^2 e_V\ . \tag{12}$$

Therefore, the velocity of water through the orifice is better described by

$$V = \sqrt{\frac{2gh}{1+e_V}}\ . \tag{13}$$

Solution of the Model

Let's turn our attention back to solution of the model. Suppose we select the long (*L*=61 cm) drain tube with a inside diameter of 0.475 cm. We fill the tank to a depth of 18 cm, allow the water to become quiescent, then start the draining process. The tank will empty in about 2 minutes. What will the model show? Let's find out.

We have several options. We can employ the Euler scheme described previously; it can be easily implemented in Excel. Or we could write our own program in a high-level language like Fortran or BASIC. Alternatively, we could use Mathcad. For the latter, we might select the fourth-order Runge-Kutta method, i.e., *rkfixed*. The actual format is as follows:

$$Z := rkfixed(h, 0, 100, 400, D)$$

In this case, we are evaluating the solution given an initial value of *h* between *t*=0 and *t*=100 s, using 400 points. The *D* represents the first derivative—that's our equation (7). The choice of the 100 s limit on *t* turns out to be insufficient. The model actually indicates that it will take about 107 s for the tank to drain out. The complete setup for this problem in Mathcad is shown on the following page, along with a crude graph of the results.

You can see immediately that we are underestimating the frictional resistance offered by the brass tube. However, an easy means of compensation has been provided. Note the assignment of:

fconst:=0.0791.

Try increasing this value by 15%, to about 0.09. How does this affect your model output?

Efflux time for a cylindrical tank with tube (d,L) of 0.475 and 61

$g := 980$

$L := 61$

$R := 0.2375$

$rho := 1$

$visc := 0.01$

$Rt := 7.775$

$fconst := 0.0791$

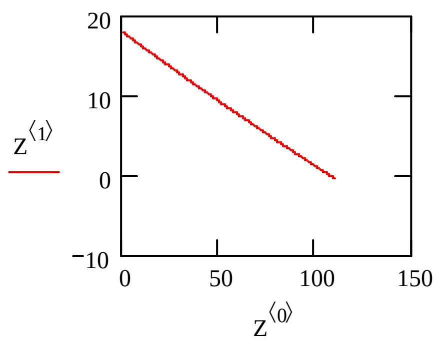
$h_0 := 18$

$$D(t,h) := \frac{-R^2}{Rt^2}\cdot\left[g\cdot(L+h)\cdot\frac{R^{\frac{5}{4}}}{(fconst\cdot L)}\right]^{\frac{4}{7}}\cdot\left(2\cdot\frac{rho}{visc}\right)^{\frac{1}{7}}$$

$Z := \text{rkfixed}(h, 0, 110, 400, D)$

Z =

	0	1
364	100.1	1.126
365	100.375	1.083
366	100.65	1.04
367	100.925	0.997
368	101.2	0.953
369	101.475	0.91
370	101.75	0.867
371	102.025	0.824
372	102.3	0.781
373	102.575	0.738
374	102.85	0.695
375	103.125	0.652
376	103.4	0.609
377	103.675	0.566
378	103.95	0.523
379	104.225	0.48



Report Requirements

A written report is required for this experiment; it will have four main components and the length can vary considerably depending upon how one decides to handle the figures (there may be as many as 16).

1) An introduction, including an accurate description of the system.

2) A graphical presentation of the experimental data for the tubes you tested (each with an informative caption).

3) A description of the model employed with a same-graph comparison (one option) to your experimental data. Please include a copy of your spreadsheet *with cell formulas*.

4) An explanation of the results with careful attention paid to any significant discrepancies.

In addition, you are expected to provide answers for the following questions (items):

•Does the Reynolds number in your trial(s) ever fall below 3000?

•What is L/d for your tube(s)?

•How much (%) would you have to increase the friction factor in order for your model and your experimental data to agree quantitatively? Demonstrate this for several of the worst-case trials.

•Would accounting for the change in kinetic energy appreciably change the behavior of your model?

•In the case of the orifice, determine the "best" value for e_v. How large is the correction that is being made to Torricelli's law?