

# TEMPERATURE PROFILES IN SOLIDS

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## INTRODUCTION

The apparatus employed in this experiment consists of three metal rods: one stainless steel rod with a diameter of 2.54 cm, one aluminum rod with a diameter of 1.27 cm, and one aluminum rod with a diameter of 2.54 cm. The rods are approximately 96 cm long and have uniform cross-section. Thermocouples are embedded at z-positions corresponding to 1.5, 4.5, 11, 17, 24.5, 32, 47, 62.5, 77.5, and 93 cm. On the right-hand side, all three rods are inserted into a brass steam chest. At  $t=0$ , low pressure, saturated steam is allowed to flow into the steam chest, instantaneously raising the temperature of the inserted rod ends to about 120 °C. The apparatus is depicted below in Figure 1. We are interested in the transient flow of thermal energy in the rods, the approach to steady-state conditions, and the effectiveness of the rods in casting off thermal energy to the surroundings. We also want to determine whether or not this experiment might be useful (if properly modified) for the determination of the thermal conductivity of metal rods.

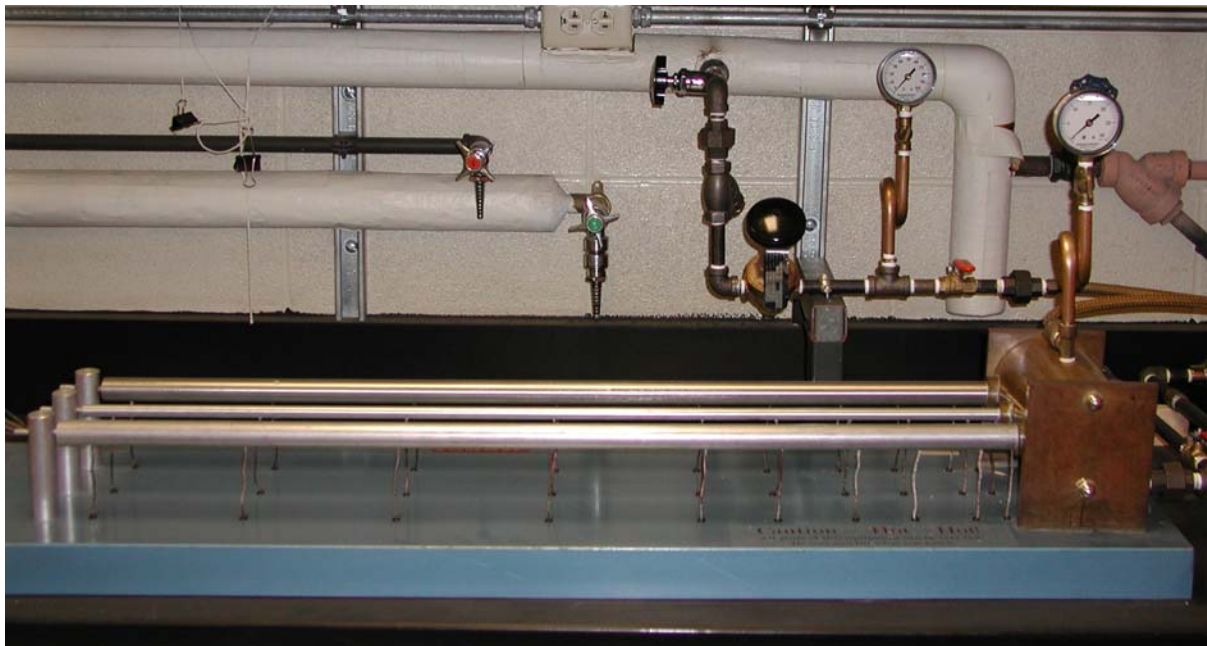


Figure 1. Temperature profile apparatus with steam chest on the right. The copper-constantan thermocouple positions are (nearly) apparent in this illustration.

## ANALYSIS OF THE PROBLEM

The energy equation in cylindrical coordinates is:

$$\rho C_p \left( \frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + \frac{V_\theta}{r} \frac{\partial T}{\partial \theta} + V_z \frac{\partial T}{\partial z} \right) = k \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + \Phi \quad (1)$$

Let's assume that the dissipation function,  $\Phi$ , is zero, that we have angular symmetry, and that the velocity vector components are zero inside the rods. We are left with:

$$\frac{\partial T}{\partial t} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] \quad (2)$$

It's apparent that there is a problem. What happens to the thermal energy arriving at the surface of the rods? Clearly it is transferred to the surroundings, mainly by free (or natural) convection. The implication, of course, is that we need eq. (1) for the fluid phase (air), and eq. (2) for the solid metal rod. Suddenly, this problem has taken on an entirely new level of complexity. We have no simple way of determining the velocity vector components for (1), so we seek a simplification that will make the problem more tractable. We begin by making a "shell" balance on a segment of the rod of length  $\Delta z$ , assuming that  $T$  is *not a function of*  $r$ ; a verbal statement of the balance is the starting point.

(rate in at  $z$ ) - (rate out at  $z+\Delta z$ ) - (loss through surface at  $R$ ) = (accumulation)

$$(\pi R^2 q_z)_z - (\pi R^2 q_z)_{z+\Delta z} - 2\pi R \Delta z h (T - T_\infty) = \rho C_p \pi R^2 \Delta z \frac{\partial T}{\partial t} \quad (3)$$

Note that the rate of loss at the surface is being described with Newton's "law" of cooling. The introduction of the heat transfer coefficient ( $h$ ) is undesirable because it is unknown, but it may permit us to find some simple solutions if  $h$  is treated as a constant. Now, divide by  $\pi R^2 \Delta z$  and apply the definition of the first derivative as  $\Delta z \rightarrow 0$ . The result is:

$$\alpha \frac{\partial^2 T}{\partial z^2} - \frac{2h}{\rho C_p R} (T - T_\infty) = \frac{\partial T}{\partial t} \quad (4)$$

It is convenient to define a dimensionless temperature,  $\theta$ :

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty}, \text{ where } T_0 \text{ is the temperature at the hot end of the rod for all } t > 0.$$

Therefore, (4) can be written:

$$\alpha \frac{\partial^2 \theta}{\partial z^2} - \frac{2h}{\rho C_p R} \theta = \frac{\partial \theta}{\partial t} \quad (5)$$

The initial condition and boundary conditions for this problem are:

$$\begin{aligned} \text{IC: } & t \leq 0, \theta = 0 \text{ (for all } z) \\ \text{BC's: } & z=0, \theta=1 \text{ and } z \rightarrow \infty, \theta=0. \end{aligned} \quad (6)$$

Pay particular attention to the second boundary condition. Obviously, this ( $z \rightarrow \infty$ ) has been chosen for mathematical rather than physical reasons. We shall return to this issue in the Appendix.

Despite its formidable appearance, eq. (5)--using the boundary conditions (6)--can be solved readily by making use of the Laplace transform. The interested reader can find details in Carslaw and Jaeger (*Conduction of Heat in Solids, Second Edition*) on page 301. This is a useful technique that can be employed to find solutions for a number of parabolic PDE's that arise in transport phenomena; however, a good table of Laplace transforms is necessary for inversion. The solution in our case is:

$$\theta = \frac{1}{2} \left[ \exp^{\sqrt{\frac{2h}{\alpha \rho C_p R}} z} \operatorname{erfc} \left( \frac{z}{\sqrt{4\alpha t}} + \sqrt{\frac{2h}{\rho C_p R}} t \right) + \exp^{-\sqrt{\frac{2h}{\alpha \rho C_p R}} z} \operatorname{erfc} \left( \frac{z}{\sqrt{4\alpha t}} - \sqrt{\frac{2h}{\rho C_p R}} t \right) \right] \quad (7)$$

Note that  $\operatorname{erfc}(\eta)$  is the complementary error function, defined by:

$$\operatorname{erfc}(\eta) = 1 - \operatorname{erf}(\eta), \text{ where the error function, } \operatorname{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta^2} d\eta.$$

A table of error function values is provided below for your use. Note that the argument of the second of the  $\operatorname{erfc}(X)$  terms can become negative; suppose, for example, that we were interested in the behavior of an aluminum rod with a diameter of 1 inch. For aluminum (alloy 1100) we find  $\alpha = 0.851 \text{ cm}^2/\text{s}$ . We arbitrarily choose  $z = 10 \text{ cm}$  and  $t = 500 \text{ s}$ . Therefore:

$$\frac{z}{\sqrt{4\alpha t}} = 0.242.$$

This means that the argument will change signs if the heat transfer coefficient,  $h$ , is larger than  $0.0000464 \text{ cal}/(\text{cm}^2 \text{ s}^\circ\text{C})$ , or in more familiar units,  $0.342 \text{ Btu}/(\text{ft}^2 \text{ hr}^\circ\text{F})$ . Of course,  $\operatorname{erf}(X)$  is symmetric, so this poses no problem. For the magnitude of  $h$ , see McCabe and Smith (*Unit Operations of Chemical Engineering*) or Bird, Stewart, and Lightfoot (*Transport Phenomena, 1st Edition*), page 413. For horizontal cylinders, the Nusselt number for free, or natural, convection is given by

$Nu = \frac{h_m d}{k} = 0.53 Ra^{1/4}$  where  $Ra = GrPr$ . For the hot end of the one inch aluminum rod, the Grashof number,  $Gr = \frac{d^3 g \beta \Delta T}{\nu^2}$ , is about  $1.1 \times 10^5$  and the Nusselt number is about 8.8. For  $z=70$  cm at long time (large  $t$ ) the Nusselt number is about 5.3. *This underscores an important limitation of our modeling effort! It is clear that  $h$  is not constant.*

Table 1. Numerical values of the error function, erf(X), and the complementary error function, erfc(X).

| X    | erf(X)  | erfc(X) |
|------|---------|---------|
| 0    | 0       | 1       |
| 0.02 | 0.0227  | 0.9773  |
| 0.04 | 0.0452  | 0.9548  |
| 0.06 | 0.0677  | 0.9323  |
| 0.08 | 0.0902  | 0.9098  |
| 0.10 | 0.1126  | 0.8874  |
| 0.20 | 0.2228  | 0.7772  |
| 0.30 | 0.3287  | 0.6713  |
| 0.40 | 0.4285  | 0.5715  |
| 0.50 | 0.5206  | 0.4794  |
| 0.60 | 0.6039  | 0.3961  |
| 0.70 | 0.6779  | 0.3221  |
| 0.80 | 0.7422  | 0.2578  |
| 0.90 | 0.7970  | 0.2030  |
| 1.00 | 0.8427  | 0.1573  |
| 1.20 | 0.9103  | 0.0897  |
| 1.40 | 0.9523  | 0.0477  |
| 1.60 | 0.9763  | 0.0237  |
| 1.80 | 0.9891  | 0.0109  |
| 2.00 | 0.9953  | 0.0047  |
| 2.50 | 0.9995  | 0.0005  |
| 3.00 | 0.99992 | 0.00008 |

We now have an analytic solution for the transient problem, assuming that the rod is infinitely long, that temperature is not a function of  $r$ , and that the heat transfer coefficient is constant. Let's focus our attention upon the latter simplification. As we noted above,  $h=f(z)$ , or more precisely,  $h=f(T)$ . To explore this, consider the case in which the temperature of the rod has virtually attained steady-state. The governing equation may now be written:

$$\frac{d^2 \theta}{dz^2} - \frac{2h}{\alpha \rho C_p R} \theta = 0, \text{ and we let } \beta = \frac{2h}{\alpha \rho C_p R}. \quad (8)$$

Consequently, a solution can be found immediately (for constant  $\beta$ ) using linear differential operator notation:

$$\theta = C_1 \exp(-\sqrt{\beta}z) + C_2 \exp(+\sqrt{\beta}z). \quad (9)$$

For our first boundary condition, we have:

$$\text{at } z=0, \theta=1, \text{ consequently } 1=C_1+C_2. \quad (10)$$

Now, what about the far end of the rod? Suppose the rod is long enough that even after a very long time, very little thermal energy is lost through the end. We could write this BC as:

$$\text{at } z=L, \quad d\theta/dz=0. \quad (11)$$

Strictly speaking, this can only be correct if we insulate the end of the rod. But it does allow us to find a very simple solution; noting that

$$C_2 = \frac{1}{1 + \exp(2\sqrt{\beta}L)} \quad (12)$$

$$\text{Consequently, we find } \theta = \frac{1}{1 + \exp(2\sqrt{\beta}L)} \left[ \frac{\exp(2\sqrt{\beta}L)}{\exp(\sqrt{\beta}z)} + \exp(\sqrt{\beta}z) \right]. \quad (13)$$

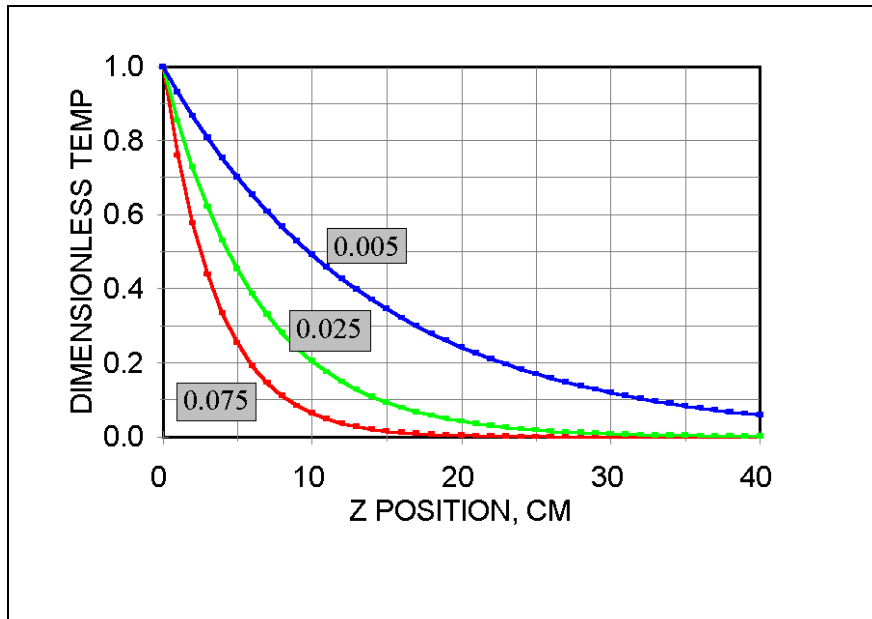


Figure 2. Typical steady-state temperature profiles for  $\beta$ 's of 0.075, 0.025, and 0.005  $\text{cm}^{-2}$ , assuming that  $dT/dz=0$  at  $z=L$  (96 cm).

This steady-state solution has an obvious flaw. The coefficient,  $\beta$  (and thus,  $h$ ), was taken as a constant, whereas we already discovered  $h=f(T)$ . Let's look at an alternative. The second-order, central difference approximation for the second derivative can be written:

$$\frac{d^2T}{dz^2} \approx \frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta z)^2} \quad (14)$$

where the index  $i$  refers to  $z$ -position. This approximation has a leading error on the order of  $(\Delta z)^2$ , so if we make the interval small enough, we can in principle obtain a good estimate for the second derivative. Consequently,

$$\frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{(\Delta z)^2} \approx \frac{2h}{\alpha \rho C_p R} \theta_i \quad (15)$$

Therefore, we can take our data for very large time ( $t$ ) and use them to estimate the heat transfer coefficient at various  $z$ -positions. Though this sounds attractive, the differentiation of *experimental data* is problematic. We will return to this difficulty in our discussion of typical results.

### ESTIMATING CONDUCTIVITY

Suppose we repeat the experiment using only the 2.54 cm diameter aluminum rod; but this time, we cover it with pipe insulation such that  $h$  is virtually zero. Naturally, the heat will flow towards the end of the rod much more rapidly. In this case the governing equation is merely:

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial z^2} \quad (16)$$

This should look very familiar; you have seen a mathematically-equivalent problem previously. Take a look at example 4.1-1 on page 115 of *Transport Phenomena* (2nd Edition). The situation of interest to us is fully analogous and a transformation variable may be employed to help us find an analytic solution; if we choose

$$\eta = \frac{z}{\sqrt{4\alpha t}} \quad (17)$$

we can show very easily that

$$\theta = \text{erfc}\left(\frac{z}{\sqrt{4\alpha t}}\right). \quad (18)$$

This provides us with a fairly simple means for the estimation of the thermal diffusivity of the 1-inch aluminum rod. However, the value we find for  $\alpha$  may not correspond to pure aluminum. See the examples provided in Table 2.

Table 2. Thermal properties of some aluminum alloys at normal ambient temperatures.

| alloy number          | density | $C_p$ , cal/(g° C) | $k$ , cal/(cm s° C) | $\alpha$ , cm <sup>2</sup> /s |
|-----------------------|---------|--------------------|---------------------|-------------------------------|
| 1100 (99+%, annealed) | 2.71    | 0.23               | 0.5305              | 0.851                         |
| 3003 (annealed)       | 2.73    | 0.23               | 0.4616              | 0.735                         |
| 2017 (annealed)       | 2.79    | 0.23               | 0.410               | 0.639                         |
| 5052                  | 2.68    | 0.23               | 0.3307              | 0.537                         |
| 7075 (annealed)       | 2.80    | 0.23               | 0.289               | 0.449                         |

### PROCEDURE

We actually run this experiment twice. In the first trial, the rods are left uncovered and all of the thermocouples are monitored. At  $t=0$ , saturated steam is allowed to flow into the steam chest, establishing a pressure of about 15 psig. This corresponds to a temperature of about 120 to 121 °C at the  $z=0$  position. Monitor the steam chest pressure and record the thermocouple outputs for time period not less than 50 minutes (we're hoping to achieve steady-state---is that very likely?). At the conclusion of the run, shut off the steam flow and cool the steam chest and the 1-inch aluminum rod to normal ambient temperature. This can be achieved fairly quickly through use of a wet sponge and a fan to promote air flow.

Install the 1-inch pipe insulation over the aluminum rod at the front of the apparatus. Monitor only the thermocouples for that rod. Admit steam to the steam chest, and maintain a pressure of about 15 psig, as done previously. This trial should have a duration of least 40 minutes. Note the significantly greater rate at which heat flows towards the far end of the rod. We will use these data in conjunction with eq. (18) to obtain an estimate of the thermal diffusivity of the aluminum rod.

### TYPICAL RESULTS

Figure 3 illustrates measured temperature profiles in the 1-inch aluminum rod at  $t=2400$  s for the case of no insulation, and at  $t=1800$  s for the insulated trial. Naturally, with the insulation installed, the conduction of thermal energy in the  $z$ -direction produces more rapid heating of the rod. We shall now use these data for purposes of demonstrating some of the necessary calculations. First, let's focus our attention upon the results of the uninsulated experiment at  $t=2400$  s.

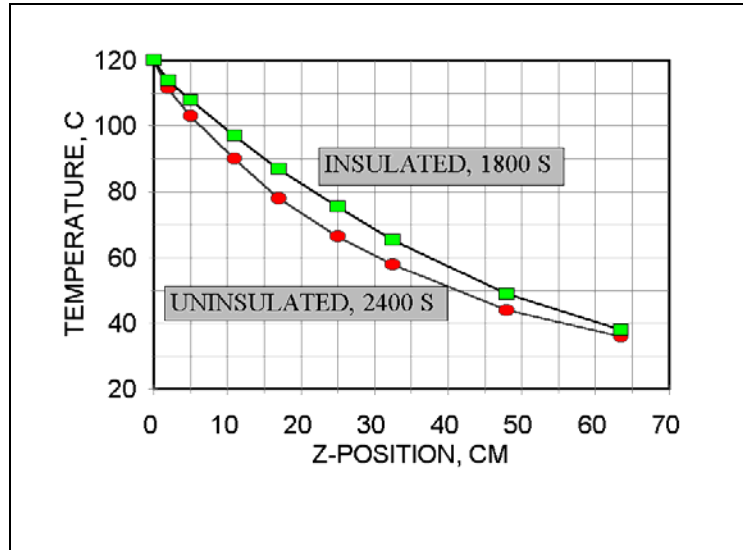


Figure 3. Characteristic experimental results for the 1-inch aluminum rod; the upper curve is for the insulated case at  $t=1800$  s, and the lower curve is from the uninsulated trial at  $t=2400$  s.

We assume that these data are approximately representative of steady-state conditions even though 40 minutes is *really not adequate*. We begin by replotting a portion of the data with an expanded scale in Figure 4; this is being done to facilitate calculation of the second derivative. For the first point, select  $z=2$  cm and let  $\Delta z$  be 1 cm. Consequently, we find (through careful interpolation):

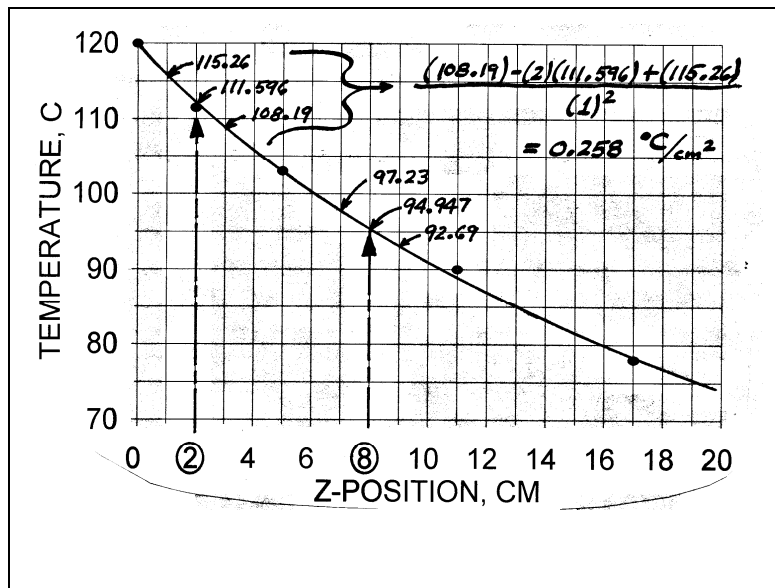


Figure 4. Illustration of the use of an expanded scale for computation of the second derivative at  $z=2$  and  $z=8$  cm. Note how important precision is with regard to the calculation!

$$\frac{d^2T}{dz^2} \approx \frac{(108.19) - (2)(111.596) + (115.26)}{(1)^2} = 0.258 \text{ } ^\circ\text{C}/\text{cm}^2 \quad (19)$$

Repeating this process for  $z=8$  cm, I find that the second derivative has fallen to approximately  $0.026 \text{ } ^\circ\text{C}/\text{cm}^2$ . You may want to try to verify that. Our dilemma is now very apparent: in order to have any chance to compute these second derivatives accurately, we must have *very precise* temperatures at the  $i-1$ ,  $i$ , and  $i+1$  points. This is very difficult to achieve with experimental data, so we will explore an alternative that may be used to advantage.

We recognize that the computation of the second derivatives will be much easier if we can develop an analytic representation of the data. There are a number of ways this can be done, and one of the more popular techniques in recent years is cubic spline interpolation. This can be achieved fairly quickly as demonstrated below. Consider an interval of  $z$ -position values as represented by  $(z_i, z_{i+1})$ ; we shall refer to this as interval "i." Now we write expressions for  $T(z)$  and its derivatives:

$$T = a_i(z - z_i)^3 + b_i(z - z_i)^2 + c_i(z - z_i) + d_i \quad (20)$$

$$\frac{dT}{dz} = 3a_i(z - z_i)^2 + 2b_i(z - z_i) + c_i \quad (21)$$

$$\frac{d^2T}{dz^2} = 6a_i(z - z_i) + 2b_i \quad (22)$$

Of course, the second derivative is only a *linear function* of  $z$ . Will that give us an adequate representation for these data? Our immediate task is to determine values for the unknown coefficients. Let's select an interval:  $z(0,20 \text{ cm})$ . It is obvious that  $d_i=120$  and that  $c_i=-5 \text{ } ^\circ\text{C}/\text{cm}$ .

In addition, we have:

$$74 - 120 = a_i(20)^3 + b_i(20)^2 - 100 \quad \text{and} \quad (23)$$

$$-1.25 = 3a_i(20)^2 + 2b_i(20) - 5 \quad (24)$$

As an aside, we note that in normal application of the cubic spline technique, end point values of the derivatives are frequently unknown. Sometimes the second derivative is simply taken to be zero at the end points. Based upon the two equations immediately above, we find:

$$a_i = -0.004125 \quad \text{and} \quad b_i = 0.2175.$$

Accordingly, the second derivative can be written as:

$$\frac{d^2T}{dz^2} \approx -0.02475z + 0.435 \quad (25)$$

Compare this result with estimates made using the finite difference approximation; at  $z=2$  and  $8$  cm, equation (25) yields  $0.385$  and  $0.237$  °C/cm<sup>2</sup>, respectively. At least our initial estimate was within an order of magnitude of these numbers!

Let's now make use of the ( $t=1800$  s) data for the insulated case to attempt to evaluate the thermal diffusivity for the 1-inch aluminum rod:

| $z$ , cm | $T$ , °C | $\theta$ | $\eta$ | $\alpha$ , cm <sup>2</sup> /s |
|----------|----------|----------|--------|-------------------------------|
| 15       | 90       | 0.677    | 0.29   | 0.372                         |
| 29       | 70       | 0.462    | 0.52   | 0.432                         |
| 47       | 50       | 0.247    | 0.81   | 0.467                         |

These rough estimates for the thermal diffusivity are at the lower end of the values presented in Table 2. Of course, this is partly the result of heat losses through the insulation itself. These findings also underscore a rather important question: Would it be preferable to use data from either smaller or larger  $t$ ? That is, would we get better estimates for the thermal diffusivity if we used data at say,  $t=500$  s, or perhaps  $t=3000$  s?

We return to the results of the uninsulated trial and think about the effectiveness of these cylindrical rods for casting off thermal energy to the surroundings. This is, of course, an example of extended surface heat transfer, similar to the fin on an automobile radiator or the cylindrical studs installed on a boiler tube. *Under steady-state* operating conditions, all of the thermal energy entering the cylinder(s) at its base ( $z=0$ ) is being transferred to the surroundings. Consequently, the total heat discarded to the air is simply:

$$Q = -\pi R^2 k \left. \frac{dT}{dz} \right|_{z=0} \quad (26)$$

Look at the  $t=2400$  s data presented in Figure 3 (recognizing that 40 minutes is not adequate to achieve steady-state); the slope at  $z=0$  is approximately  $-5$  °C/cm. Consequently, we find

$$Q = 3.1416(1.613)(0.46)(5) = 11.66 \text{ cal/s or } 167 \text{ Btu/hr}$$

for the 1-inch aluminum rod. This result raises some important questions regarding extended surface heat transfer. Which material is more effective, (type 304) stainless steel, or aluminum? Which diameter is preferred, 1 inch or 0.5 inch? Is the cylindrical shape a good one for this purpose, and if not, what shape would be better?

## SPECIFIC REPORT COMPONENTS

The principal elements of the report (results section) for this experiment include:

- 1) Experimental temperature profiles for all three rods at short, intermediate, and long (steady-state?) times.
- 2) An assessment of how well the transient model, eq. (7) describes the observed behavior.
- 3) Computations of the second derivative (wrt z) at large (steady-state) time and an evaluation of whether or not  $h=f(z)$ . Carry this out for the 1-inch aluminum rod only.
- 4) Evaluation of the steady-state model, eq. (13), for the 1-inch aluminum rod.
- 5) Rates of heat dissipation (at maximum t) for all three rods.
- 6) Estimation of the thermal diffusivity of the 1-inch aluminum rod obtained using the data from the insulated trial.

#### LITERATURE CITED

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#### APPENDIX

This section is provided for the student interested in expanding his/her capabilities with regard to solution of problems in transport phenomena. We begin by rewriting the governing equation (4) in finite difference form; let i be the spatial (z) index and let j be the time index:

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta t} \approx \alpha \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta z)^2} - \frac{2h}{\rho C_p R} (T_{i,j} - T_\infty)$$

Note that the only j+1 index appears in the time derivative on the left-hand side of the equation. All of the other T values are on the old (previous) time-step row, and are therefore known. This

provides a very simple *explicit* algorithm for solution of the partial differential equation. We merely multiply by  $\Delta t$  and add  $T_{i,j}$  to both sides. Keep in mind, however, that we are limited to small time-step sizes (to ensure numerical stability). Let's develop a little program for the solution of this problem:

```
#DIM ALL
  REM *** Computation of T(z,t) for an aluminum rod heated at one end
  REM *** Program written for ChE 535 by Larry Glasgow
  GLOBAL dt,dz,tlim,alf,rho,cp,R,Tair,h,i,ttime,z,d2tdz2 AS SINGLE
FUNCTION PBMAIN
  DIM T(49,2) AS SINGLE
  dt=1:dz=2:tlim=3000:alf=0.55:rho=2.71:cp=0.23:R=1.27:Tair=27:h=0.0001
  OPEN "c:Tprofil3.dat" FOR OUTPUT AS #1
  T(1,1)=120
  FOR i=2 TO 49
    T(i,1)=27
  NEXT i
  ttime=0
50 REM *** explicit computation of T
  FOR i=2 TO 48
    d2tdz2=(T(i+1,1)-2*T(i,1)+T(i-1,1))/dz^2
    T(i,2)=dt*alf*d2tdz2-2*dt*h/(rho*cp*R)*(T(i,1)-Tair)+T(i,1)
  NEXT i
  ttime=ttime+dt
  PRINT ttime,T(2,2),T(10,2),T(20,2)
  REM *** flop time values
  FOR i=2 TO 48
    T(i,1)=T(i,2)
  NEXT i
  IF ttime>tlim THEN 200 ELSE 50
200 REM *** output results
  FOR i=1 TO 49
    z=(i-1)*dz
    WRITE#1,z,T(i,1)
  NEXT i
  CLOSE:END

END FUNCTION
```

Note that in this program,  $T(z=L)$  is assigned the ambient value; see line 100. While this boundary condition is appropriate for an infinitely long rod, it may not be valid for our case. We have two other options:

No heat flow through end of rod: at  $z=L$ ,  $\frac{dT}{dz} = 0$ .

Equate fluxes at rod end: at  $z=L$ ,  $-k \frac{dT}{dz} = h(T_{z=L} - T_\infty)$

It is a valuable learning experience to incorporate these BC's into the program. How do the different boundary conditions affect the solution?

Sample profiles computed with this program are shown in Figure 5 for  $t=1000$  s and  $t=3000$  s. Note that at 3000 s, the thermal energy has reached the far end of the aluminum rod. This does not occur in the experiment, which suggests that the selected value for the heat transfer coefficient,  $h$ , is too low.

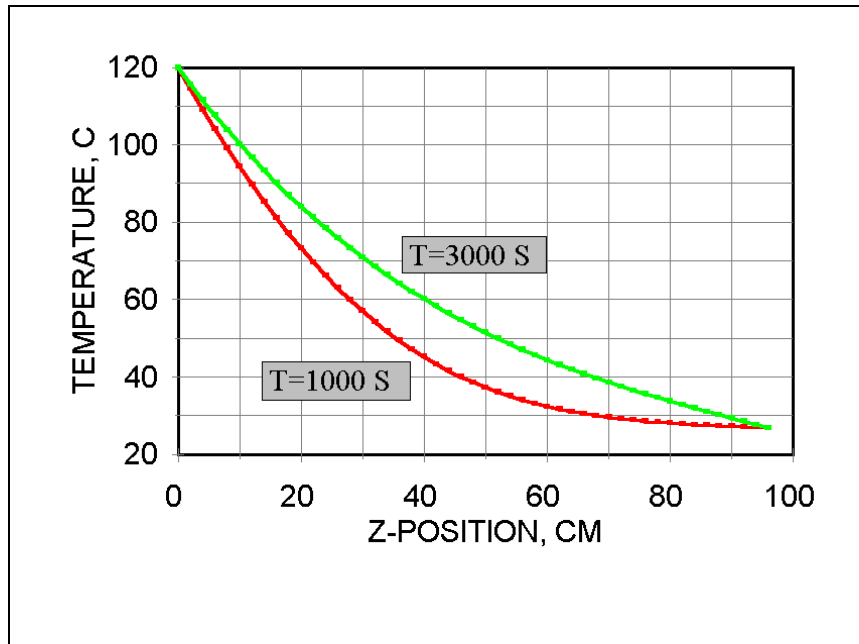


Figure 5. Computed axial temperature profiles for the 1-inch aluminum rod using the following values for  $h$  and  $\alpha$ :  $0.0001 \text{ cal}/(\text{cm}^2\text{sC})$  and  $0.55 \text{ cm}^2/\text{s}$ , respectively.